

A common scenario for an small vacuum energy and long lived super heavy dark matter

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Abstract

A toy model originating super heavy dark matter and an small vacuum density energy, of the order of the one measured in the present era is constructed. This is obtained by considering a hidden sector with an axion like particle associated to an extremely weak interaction together with a super massive Higgs like boson. The axion acts as a false vacuum, and the hidden Higgs may be created in the early universe. By employing a crude estimation we suggest that the mean lifetime of this hidden Higgs is larger than the age of the universe. We argue that this particle ac as a component of the dark matter at present times. The approach to the vacuum energy problem presented here is a quintessence like mechanism, in which it is assumed that the true vacuum density energy is zero for some reason, except for the contribution of the light axion.

1. Introduction

The energy density of the universe at present times seems very close to the critical one $\rho_c \sim 10^{-47} \text{GeV}$ [1]. This value is not explained by the current understanding of QFT since the scale of supersymmetry breaking and of the quark condensate are around 55 and 43 orders of magnitude higher than the critical one, respectively. Thus there are two questions to be answered, why the vacuum density energy is so small and why it is so close to the critical one. These problems arguably demands new physics, even with the inclusion of supersymmetry.

An approach for explaining the smallness of the vacuum energy are the cancelation mechanisms [2]-[4]. In these scenarios an initial energy density is present, together with an unknown component which contributes to the this density with opposite sign, in such a way that for large time the total density becomes very small. An alternative point of view was suggested in [5]-[6] where it was argued that the de Sitter space itself suffers an adiabatic catastrophe rendering it unstable. This situation was also considered in [7]. Additionally these references suggest that there are infrared effects which result in an effective screening of the vacuum energy, thus rendering it very small. Nevertheless the same technical results were obtained in [8] but the interpretation given there is that there is no instability. This is a topic of discussion at present.

A completely different scenario for the vacuum energy is given by the quintessence models [9]. In this setup the vacuum energy is associated with an slowly rolling scalar field which is not at the minimum of its potential at the present times. The difference between quintessence and cancelation mechanisms is that in the first an initial energy is assumed, while in the second no vacuum energy is present except for the scalar field. To ignore the QFT vacuum density from the very beginning may sound unsatisfactory for some, but it was indicated in [10] that there are different string theories with axion like pseudoscalar particles [11] miming the characteristics of the quintessence. Additionally this reference present a quantity of models in which the true vacuum energy is zero. In this context the idea to have only a small contribution due to a light scalar field may sound plausible. An axion

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pseudoparticle is in fact one of the ingredients of the present work. Incidentally, axions play an important role in modern theoretical cosmology [12] and axion emission by cosmic strings in a more theoretical setup was considered long time ago in [13].

Another actual question is the existence of super-heavy dark matter at present times [14]. In fact super-heavy particles were though as possible candidates to explain the origin of ultra high energy cosmic rays (UHECR) [16]-[17], i.e, cosmic rays with energy saturating the GKZ bound 10^{11}GeV [15]. These models include the existence of gravitationally created super heavy particles (with mass of the order of 10^{13}GeV) during inflation or topological defects originated by non thermal phase transitions. Since these objects are created in the early universe they should have a time life of the order of 10^{10} years in order to decay at present times [18]-[20]. The AUGER project seems to discard the existence of UHECR above the GKZ bound [21]. Still, to study of mechanisms for long lived super massive particles it may be of importance in the context of dark matter physics, even with the inclusion of supersymmetry [22]. It is not a trivial task to find scenarios with long lived super heavy particles since, a priori, the heavier a particle is, the more probable the decay seems to be. The stability may be warranted by symmetry protection mechanism [23] or because they weakly interact with ordinary matter. The last type type of particles are known as WIMP (weakly interacting massive particles).

From the point of view of the dark matter problem, it is natural to postulate the existence of a hidden sector of particles. This is an old concept which was introduced Salam, Lee and Yang in order to save the parity violation [24]-[25]. In fact there is a present line of though which suppose the existence of mirror particle sectors in order to avoid the left right asymmetry of particle physics [26]. Cosmological implications of hidden sectors has been studied for instance in [27] and hidden sectors with Higgs like field particles are considered in [30]. Additionally hidden Higgs candidates for dark energy and dark matter are considered in [31]-[35]. But the hidden Higgs that we will consider here is more massive that the ones in those references.

In the present work we consider a hidden sector with a Higgs like super massive particle whose mean lifetime is of the order of, or even bigger than, the age of the universe. This model also contains is a very light axion pseudoscalar. This axion is under the influence of an effective potential originated by a very weak interaction between the hidden sector with ordinary matter and thus acquires an small mass. The presence of this axion produce an energy density of the order of the critical one.

The organization of this paper is as follows. In section 2 general properties of the classical axion model are discussed. Section 3 contains preliminary explanations which are necessary in order to fix the scales of the model. In particular, it is deduced that the interactions of the hidden sector with the ordinary one should be of gravitational order. Based in this observations we propose our unified toy model in section 4. In section 5 we argue that the pseudoscalar axion of the presented model plays the role of a quintessence field giving the critical energy density ρ_c , and that the massive Higgs like particle is long lived with a mean time life of the order of the age of the universe if its mass $m_h \sim 10^{13}\text{GeV}$. Lighter masses results in an increase of the mean lifetime. Section 6 contains the discussion of the presented results in more detail. We should remark that our work is in the spirit of [52] but the technical details considered here are different from those of that reference.

2. A brief review of the axion in QCD

As is well known, in ordinary QCD it should be included the θ term associated with the instantons of the theory [37]-[38]. This term violates CP invariance. For massless QCD the transformation

$$\psi \rightarrow e^{i\gamma_5\alpha}\psi, \quad (2.1)$$

on the fermions of the theory will be a classical symmetry of the lagrangian. At quantum level there is an anomaly in the chiral current J_5^μ

$$\partial_\mu J_5^\mu = \frac{g^2}{16\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu},$$

related to the gluon field strength $G_{\mu\nu}^a$. For this reason if the fermions were massless the chiral transformation would induce the following one

$$\theta \rightarrow \theta - 2\alpha,$$

to the θ parameter. This means that for massless QCD all the theories with different θ will be equivalent. It is the mass of the fermions which spoils the chiral symmetry and simultaneously the CP invariance. Experimental bounds on the value of θ shows that $\theta < 10^{-9}$. This value does not satisfy the majority, who considers the introduction of such small parameter in the theory as unnatural. For this reason in [11] there was introduced an alternative to explain this naturalness problem in which the θ parameter is considered as a dynamical field, the axion, which runs to the value zero no matter which initial value is. The lagrangian describing the axion a and its interaction with the gluons is

$$L_a = L_{QCD} + L_k(a) + (\theta + \frac{a}{f_a}) \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu},$$

with f_a an axion constant and $L_k(a)$ its kinetic term. This corresponds to an effective $\bar{\theta}$ term with

$$\bar{\theta} = \theta + \frac{a}{f_a}.$$

By shifting the field $a \rightarrow a - f_a \theta$ the θ parameter can be discarded. Thus the theory will be CP invariant if there is something that forces the axion to take the value $a = 0$. This is precisely what happens. The axion is under the influence of an effective quantum potential $V(a)$ due to the effect of the quarks and gluons inside the Feynmann path integral. By definition this potential is given by

$$\exp(- \int d^4x V(a)) = \int [DA_\mu] \prod_i [Dq_i][D\bar{q}_i] \exp \left[- \int d^4x \left(L_{QCD} + \frac{\alpha_s}{8\pi f_a} a G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \right) \right],$$

and its explicit expression has been presented for instance in [48]

$$V(a) \sim f_\pi^2 m_\pi^2 \left[1 - \cos\left(\frac{a}{f_a}\right) \right].$$

The last formula implies that the minima is at $a = 0$ and this solves the CP problem. Additionally it implies that the axion possess a mass of the order

$$m_a \sim \frac{f_\pi m_\pi}{f_a}. \quad (2.2)$$

Here f_π and m_π are the pion coupling constant and its mass respectively. Nevertheless these results should be supplemented with the CP violating terms of the weak interaction, and this results in a very tiny but non zero value of θ .

There are several axion scenarios discussed in the literature [39]-[49]. In several of these setups the axion does not interact directly with the quarks, and the coupling $aG\tilde{G}$ is interpreted of an effective interaction due to a hidden sector. For instance the diagram of the figure 1, for which the triangle is composed by a heavy quark, gives rise of an effective interaction of the form $f_a^{-1} aG\tilde{G}$ in which the axion parameter f_a is a function of the mass of this hidden quark.

Let us consider, following [41] and [49], the addition of the following lagrangian

$$L_{add} = i\bar{\psi}\hat{D}\psi - (\delta\varphi\bar{\psi}_R\psi_L + \delta^*\varphi^*\bar{\psi}_L\psi_R) + (\partial_\mu\varphi^*)(\partial_\mu\varphi) + m^2\varphi^*\varphi - \lambda(\varphi^*\varphi)^2, \quad (2.3)$$

to the ordinary QCD lagrangian. The parameters λ , m and δ are constants to be determined and the term $i\bar{\psi}\hat{D}\psi$ includes the kinetic energy of the new quark ψ and its coupling with the gluons. Since

the new scalar field acquires a non zero expectation value $|\langle \varphi \rangle| = \varphi_0 = m/\sqrt{2\lambda}$, the model describe an scalar field with mass $m/\sqrt{2\lambda}$ and a massless pseudoscalar a , defined by the equality

$$\varphi = \varphi_0 \exp\left(\frac{ia}{\varphi_0\sqrt{2}}\right).$$

This pseudoscalar is identified as the axion and it is the Goldstone boson associated to the breaking of the $U(1)$ symmetry

$$\psi \rightarrow e^{i\gamma_5\alpha}\psi, \quad \varphi \rightarrow e^{-2i\alpha}\varphi, \quad (2.4)$$

present in the lagrangian (2.3). This field does not interact directly with the light quarks and with the gluons, but acquires an effective interaction with the last due to the diagram of the figure 1. The resulting interaction is

$$\frac{\alpha_s^2}{8\pi\sqrt{2}\varphi_0} a G_{\mu\nu}^a \tilde{G}^{a\mu\nu}.$$

and from here it follows that $f_a = \sqrt{2}\varphi_0$. Since the mass of the heavy quark is proportional to f_a , the heavier the quark is, the lighter the axion will be.

The axion mechanism for solving the CP problem in QCD seem to be ruled out by the experiments but nevertheless this scenario will be helpful for constructing our model.

3. Preliminar explanation

As we have mentioned, the model we propose is intended to describe the existence of a very small vacuum energy and simultaneously the existence of long lived super heavy dark matter. For simplicity we will consider a model with a Higgs like boson \tilde{h} and an axion like scalar pseudoboson \tilde{a} which acquires a small mass due to a mixing with the ordinary pion. The boson \tilde{h} will be a component of the long lived dark matter at the present while the axion like particle mass generate an effective vacuum density. Our approach is a quintessence like model, in which it is assumed that the energy density is zero except for the presence of this axion \tilde{a} .

The Hubble parameter is related to the critical density ρ_c and the Newton constant by the Friedmann equation

$$H^2 = \frac{8\pi}{3} G_N \rho_c. \quad (3.5)$$

This is a classical equation. Besides, the actual Hubble constant satisfy the following relation with the pion mass

$$H_0 = G_N m_\pi^3. \quad (3.6)$$

This is written in natural units, and a factor c^2/\hbar^2 is in the right hand for other unit system. Thus this is an intriguing quantum relation. It is interesting to note that this relation involves the strong and the gravitational scales. One may wonder if this is a simple coincidence or if it is indicating something deeper. We will adopt a dynamical vision and interpret the vacuum energy originated by a light particle with mass $m_{\tilde{a}} \sim H_0$. This condition insures the particle not to be in the minimum of the potential, thus acting as a small energy density. The relation (3.5) shows that

$$m_{\tilde{a}}^2 \sim \frac{M^4}{M_{Pl}^2} \quad (3.7)$$

with a mass scale $M = 10^{-2} - 10^{-3} \text{eV}$. Additionally (3.6) shows that

$$m_{\tilde{a}} \sim \frac{m_\pi^3}{M_{Pl}^2}. \quad (3.8)$$

The last relation shows that $m_{\tilde{a}} \sim 10^{-32}\text{eV}$. The mass scale M is then

$$M \sim \left(\frac{m_{\pi}}{M_{Pl}} \right)^{\frac{1}{2}} m_{\pi}.$$

The task is to find a model for which these numerical relations are satisfied and which simultaneously give rise to superheavy dark matter.

The relation (3.8) resembles (2.2) since it relates the mass of the pseudoscalar field \tilde{a} with the pion mass m_{π} . This may suggest that \tilde{a} to be a axion pseudoscalar related to a symmetry breaking mechanism. This idea is implemented for instance in [50]. In this reference a fermion, which we name as hidden neutrino $\tilde{\nu}$, coupled to an scalar field in the same fashion as the axion model (2.3) is presented. The world hidden is due to the fact that interacts very weakly with ordinary matter to be observed directly. The breaking of the $U(1)$ symmetry (2.4) give rise to an axion \tilde{a} . Due to a weak interaction with ordinary matter this axion is under the influence of an effective potential and acquires a mass $m_{\tilde{a}}$ and a coupling constant $f_{\tilde{a}}$, but associated to this very weak interaction. In fact one can estimate the value of this coupling constant. By a reasoning totally analogous to the usual axion it follows that \tilde{a} acquires a potential of the form

$$V(\tilde{a}) \sim M^4 \left[1 - \cos \left(\frac{\tilde{a}}{f_{\tilde{a}}} \right) \right].$$

The condition $V(\tilde{a}) \sim \rho_c$ fixes the value $M \sim 10^{-3}\text{eV}$ found previously. The axion mass is given by

$$m_{\tilde{a}} = \frac{M^4}{f_{\tilde{a}}^2},$$

and from the value $m_{\tilde{a}} \sim 10^{-32}\text{eV}$ it is found that $f_{\tilde{a}} \sim 10^{19}\text{GeV}$. This suggest that the corresponding interaction is very weak, and it may be even of the gravitational order [51]. In fact the idea of an interaction of gravitational order between the hidden and ordinary sector has been developed in [36] in the context of D-branes.

It is not difficult to see that the relation (3.8) can be rewritten in terms $f_{\tilde{a}}$ as

$$m_{\tilde{a}} \sim \frac{m_{\pi}}{M_{Pl}} \frac{f_{\pi}}{f_{\tilde{a}}} m_{\pi}. \quad (3.9)$$

The meaning of this relation seems transparent. It is analogous to (2.2) but corrected by a factor m_{π}/M_{Pl} . This correction appears due to the conversion from the strong interaction to a very weak one. Being this axion so light, the mass of the hidden neutrino $\tilde{\nu}$ should be very large. In fact it is given by $m_{\tilde{\nu}} \sim f_{\tilde{a}}$ and thus it is of the order of the Planck mass, i.e, 10^{19}GeV .

4. Proposed scenario

We start with the assumption that there is an scalar component X in the superheavy dark matter with an interaction with the ordinary sector weak enough to ensure that its mean lifetime is bigger of the age of the universe. This particle should be created in the early universe, where creation of particles with mass of the order of the GUT scale ($10^{15} - 10^{16}\text{GeV}$) may took place. The scenario discussed in section 3 do possess massive particles and very weak interactions and it will be useful in the following, up to some modifications.

The simplest decay channel is the one in the diagram of the figure 2. The resulting amplitude is calculated in all the standard textbooks and it is seen that in order to obtain the desired mean time life it is needed an interaction weaker that the gravitation. To avoid going beyond the Planck scale we will consider instead an X which does not have a direct coupling with the known standard model

particles but acquires an effective interaction due to the diagram 3, which is analogous to the axion diagram 1. The triangle is composed by a fermion F which is heavier than X , otherwise the X may decay in two of these fermions with a diagram analogous to diagram 2, and the mean lifetime will be short. The massive fermions of the triangle interacts very weakly with the standard model particles through a boson E , which we choose of spin 1 for sake of simplicity. Additionally, we will consider that X is scalar, for the same reason.

In more concrete terms we consider an scalar Φ coupled to two fermions, which we will call h-electron and h-neutrino. We denote them as \tilde{e} and $\tilde{\nu}$, the tilde is in order to distinguish them with the ordinary leptons. By analogy with the standard model we propose an $SU(2)_L$ gauge symmetry with gives three vector bosons \tilde{W}_i with $i = 1, 2, 3$. Additionally we assume that the fermions have also right components, taken as singlets, and that the lagrangian corresponding to the right component of the h-neutrino has a $U(1)$ symmetry, which is broken spontaneously to the Planck scale and give rise to an axion. The coupling between the scalar field Φ and the neutrino is given by

$$L = \Phi \psi_{\nu_R}^T C \psi_{\nu_R},$$

C being the charge conjugation matrix. The axion is identified through the equality

$$\Phi = f_a e^{i \frac{\tilde{\Phi}}{f_a}}.$$

The current associated to the symmetry has an anomaly due to the presence of fields of an interaction of the gravitational order. The mass of the right component of the h-neutrino is $m_{\tilde{\nu}_R} \sim f_a \sim 10^{19} \text{ GeV}$ and $m_a \sim 10^{-32} \text{ eV}$ from the discussion done previously. This axion is acting as a small vacuum energy which is present at this era. The vector bosons acquire a mass $2m_{\tilde{W}} = gv$ due to the Higgs mechanism, and the h-Higgs boson acquires the mass $m_{\tilde{h}} = 2\lambda^{\frac{1}{2}}v$. The minimum of the Higgs doublet \tilde{H}

$$|< \tilde{H} >| = \begin{pmatrix} 0 \\ v \end{pmatrix},$$

and the coupling constant λ appearing in the potential for the Φ particle are free parameters, and therefore one can give any mass to these two particles.

In addition we need a coupling of the form of the diagram of the figure 2 but with the output particles heavier than the Higgs. These diagram will allow us to put the fermion triangle in the leading diagram for the decay such as in the diagram of the figure 3. For this purpose we introduce a Dirac coupling of the Higgs like doublet \tilde{H} with the h-electron and the h-neutrino given by

$$f_e \tilde{l}_L \tilde{H} \psi_{e_R} = f_e \tilde{\psi}_{e_L} \frac{v}{\sqrt{2}} \psi_{e_R} + f_e \tilde{\psi}_{e_L} \tilde{h} \psi_{e_R},$$

$$f_\nu \tilde{l}_L \tilde{H}^c \psi_{\nu_R} = f_\nu \tilde{\psi}_{\nu_L} \frac{v}{\sqrt{2}} \psi_{\nu_R} + f_\nu \tilde{\psi}_{\nu_L} \tilde{h} \psi_{\nu_R}.$$

Here l_L the doublet composed by the left components of the h-electron and h-neutrino and $f_{e,\nu}$ are constant to be determined. Due to this coupling the fermions acquire a mass $m_{\tilde{\nu},e} = v f_{\tilde{\nu},e}$ and there appears the vertex of the diagram 4. We identify then the hidden Higgs with the particle X previously mentioned.

The mass of both components of the electron are m_e while the neutrino acquire a mass matrix with different eigenvalues, one due to the Dirac coupling and the other due to the coupling with Φ . Its mass matrix is

$$M_{\tilde{\nu}} = \begin{pmatrix} 0 & m_{\tilde{\nu}} \\ m_{\tilde{\nu}} & m_{\tilde{\nu}_R} \end{pmatrix}, \quad m_{\tilde{\nu}_R} \sim f_a,$$

and since $m_\nu \ll m_{\tilde{\nu}_R}$ then the approximate eigenvalues are

$$\lambda_1 \simeq m_{\tilde{\nu}_R}, \quad \lambda_2 \simeq \frac{m_{\tilde{\nu}}^2}{m_{\tilde{\nu}_R}} \quad (4.10)$$

with the eigenvectors

$$\begin{aligned} \psi_\nu^1 &= \frac{1}{\sqrt{1 + \left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)^2}} \psi_{\nu_L} + \frac{\left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)}{\sqrt{1 + \left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)^2}} \psi_{\nu_R}, \\ \psi_\nu^2 &= \frac{1}{\sqrt{1 + \left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)^2}} \psi_{\nu_L} - \frac{\left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)}{\sqrt{1 + \left(\frac{m_{\tilde{\nu}_R}}{m_{\tilde{\nu}}}\right)^2}} \psi_{\nu_R}. \end{aligned}$$

In the cases when $m_\nu \ll m_{\tilde{\nu}_R}$ it follows that $\psi_\nu^1 \sim \psi_{\nu_R}$ and $\psi_\nu^2 \sim \psi_{\nu_L}$. This is an example of a see-saw mechanism.

The last task is to give values to the free parameters of our model. We took $f_{e,\nu} \sim g \sim 1$ and the mass of the vector bosons of the order of the GUT scale, which gives $v = 10^{16} \text{ GeV}$. This expectation value gives $m_{e,\nu} \sim 10^{16} \text{ GeV}$, also of the order of the GUT scale. Since the left and right electron has the same mass, it is safe to say that the electron has a mass of the GUT. But the neutrino has two eigenvalues (4.10), the first is of the order of the Planck mass and the other, with this value $m_{\tilde{\nu}} \sim 10^{16} \text{ GeV}$ results in $\lambda_2 \sim 10^{13} \text{ GeV}$. Still the parameter λ is not defined, which means that the mass of the Higgs is arbitrary. We are going to request that this mass is smaller than the minimal mass of the fermions, in order to diagram 4 to be the principal decay channel. This fix $m_h < \lambda_2 \sim 10^{13} \text{ GeV}$ and thus $\lambda < 10^{-3}$. But although m_h is smaller than λ_2 , we are going to require both to be of the same order of magnitude. This is in order to keep the hidden Higgs as massive as possible.

5. Hidden Higgs lifetime

The leading decay mechanism for the hidden Higgs is the diagram of the figure 3, which corresponds to the following amplitude

$$\begin{aligned} M &= \frac{g_1(g_2)^2}{2\pi^8} \sum_{ijk} \int \int dk_1^4 dk_2^4 \frac{\text{tr}\{[\not{p}_1 - \not{k}_1 - \not{k}_2 + m_{F_i} I] \gamma^\mu [\not{p}_2 - \not{k}_1 - \not{k}_2 + m_{F_j} I] (\not{k}_2 + m_{F_k} I) \gamma^\nu\}}{[(p_1 - k_1 - k_2)^2 - m_{F_i}^2][(p_2 - k_1 - k_2)^2 - m_{F_j}^2]} \\ &\quad \frac{\eta_{\mu d} \eta_{\nu \xi} (\gamma^d)^{mj} (\gamma^\xi)^{ek} \bar{\psi}_e \psi_m (\not{k}_1 + m I)_{jk}}{[k_2^2 - m_{F_k}^2][k_1^2 - m^2](p_2 - k_1)^2(p_1 - k_1)^2}. \end{aligned} \quad (5.11)$$

Here m can be a typical mass of a light fermion in the usual standard model and m_{F_i} is the mass of the fermions on the triangle which, as we discussed, can take the values m_e , λ_1 or λ_2 . Also g_1 can be f_e or f_ν and g_2 is the coupling constant between the fermion and the gauge field. We have discussed in the previous section that this interaction may be of gravitational order, but in order to make a quick estimate we are going to consider that the interaction between the hidden and ordinary sector is mediated by an spin 1 gauge field. The value of g_2 will be fixed after the estimation of the integral. The first step is to calculate the graph as if the internal line was composed by one fermion, and the mixing effect will be consider latter on.

At first sight, the integral seems to have a logarithmic divergence. The numerator is an expression of degree four, the denominator is of degree twelve and the integration is in eight variables. But if the trace is expanded it is found that

$$\text{tr}\{[\not{p}_1 - \not{k}_1 - \not{k}_2 + m_{F_i} I] \gamma^\mu [\not{p}_2 - \not{k}_1 - \not{k}_2 + m_{F_j} I] (\not{k}_2 + m_{F_k} I) \gamma^\nu\} = (\not{p}_1 - \not{k}_1 - \not{k}_2)_\xi (\not{p}_2 - \not{k}_1 - \not{k}_2)_\eta$$

$$\times \text{tr}(\gamma^\xi \gamma^\mu \gamma^\eta \gamma^\delta \gamma^\nu) + m_F (\not{p}_1 - \not{k}_1 - \not{k}_2)_\xi (\not{p}_1 - \not{k}_1 - \not{k}_2)_\eta \text{tr}(\gamma^\xi \gamma^\mu \gamma^\eta \gamma^\nu) + \dots$$

where the dots denote terms with lower powers in the momentum. The first term of this expansion is zero, since it is multiplied by the trace of a product of an odd number of gamma matrices. This means that the numerator is cubic and not quartic in the external momenta, and the integral is convergent instead of logarithmically divergent.

In order to calculate the decay width Γ and the mean lifetime $\tau = \Gamma^{-1}$ of the boson \tilde{h} we choose a coordinate system in which the Higgs is at rest

$$q = (m_h, 0, 0, 0), \quad p_1 = (p_{10}, p_{1x}, 0, 0), \quad p_2 = (p_{20}, p_{2x}, 0, 0).$$

Energy-momentum conservation implies

$$p_{1x} = -p_{2x}, \quad p_{1,20}^2 - p_{1,2x}^2 = m^2, \quad p_{10} + p_{20} = m_h,$$

and it may be shown that

$$p_{10} = p_{20} = \frac{m_h}{2}.$$

The decay width Γ is obtained by integrating

$$d\Gamma = \frac{1}{m_h} \sum_{s_1 s_2} |M|^2 \frac{d^3 p_1 d^3 p_2}{(2\pi)^2 E_1 E_2} \delta^4(p_1 + p_2 - q),$$

with respect to p_2 and p_1 , which gives

$$\Gamma = \frac{1}{m_h} \int \frac{d^3 p_1 \delta(\sqrt{p_1^2 + m^2} - \frac{q}{2})}{(\sqrt{p_1^2 + m^2})^2} \sum_{s_1 s_2} |M|^2.$$

By further defining $u = \sqrt{p_1^2 + m^2}$ it follows that

$$\Gamma = \frac{1}{m_h} \int \frac{du \sqrt{u^2 - m^2}}{u} \delta(u - \frac{q}{2}) \sum_{s_1 s_2} |M|^2,$$

which results in

$$\Gamma = \frac{\sqrt{m_h^2 - m^2}}{m_h^2} \sum_{s_1 s_2} |M|^2 \simeq \frac{1}{m_h} \sum_{s_1 s_2} |M|^2.$$

In the last step we have used that $m_h \gg m$. The last task is to calculate explicitly (5.11) and insert it into the last formula and after that to perform the sum over the polarizations of the particles resulting from the decay. This is a complicated task as we do not know of the result of the eight dimensional integral explicitly. Therefore we will make just an estimation. Clearly, when the square of M is considered, an expression of the form $[\bar{\psi}^{s_1}(p_1) \gamma_\xi (\not{k}_1 + mI) \gamma_\mu \psi^{s_2}(p_2)]$ will appear. Since this expression is a number and the gamma matrices are hermitian we have that

$$[\bar{\psi}^{s_1}(p_1) \gamma_\xi (\not{k}_1 + mI) \gamma_\mu \psi^{s_2}(p_2)]^* = [\bar{\psi}^{s_1}(p_1) \gamma_\xi (\not{k}_1 + mI) \gamma_\mu \psi^{s_2}(p_2)]^\dagger = \bar{\psi}^{s_2}(p_2) \gamma_\mu (\not{k}_1 + mI) \gamma_\xi \psi^{s_1}(p_1).$$

With a help of this expression and the equality

$$\sum_{s_1} [\bar{\psi}^{s_1}(p_1)]^A [\psi^{s_1}(p_1)]^B = (\not{p}_1 + mI)^{AB},$$

one can see that $\sum_{s_1, s_2} |M|^2$ is an expression which contains $p_{1,2}^2 = m^2$ and $p_1 \cdot p_2 \sim m_h^2$. We will replace all these expressions by the biggest mass scale, namely m_h^2 , since we are interested to obtain a lower bound of a lifetime and thus an upper bound for the width Γ . This means that

$$|M|^2 \sim \alpha_1 \alpha_2^4 m_h^2 I^2(m, m_f, m_h)$$

with $4\pi\alpha_{1,2} = g_{1,2}^2$ and $I(m, m_f, m_h)$ a complicated eight dimensional integral with respect to the internal variables k_1 and k_2 . Then

$$\Gamma \sim \alpha_1 \alpha_2^4 m_h I^2(m, m_f, m_h).$$

Since in natural units Γ has energy dimension and m_h is multiplying the whole expression, it follows that $I(m, m_f, m_h)$ is dimensionless. This means that we may represent it as

$$I(m, m_f, m_h) = f(y),$$

with

$$y = \left(\frac{m}{m_f}\right)^\alpha \left(\frac{m}{m_h}\right)^\beta \left(\frac{m_f}{m_h}\right)^\gamma.$$

Actually we can consider any power $x = y^k$ of this variable and fix it such that $x = mh(m_f, m_h)$. Then $f(y) = F(x)$. If we introduce a number $0 < \kappa < 1$ we can consider two possibilities, namely

$$x_1 = \frac{m}{m_f^\kappa m_h^{1-\kappa}},$$

or

$$x_2 = \frac{m_h}{m_f^\kappa m^{1-\kappa}}.$$

The reason for which the mass m_f is surely on the denominator is that one expects that

$$\lim_{m_f \rightarrow \infty} I(m, m_f, m_h) = 0,$$

in other words, the decay should be slower when the mass m_f in the triangle is bigger. Thus $F(x_{1,2})$ should satisfy $F(0) = 0$. But the limits related to m and m_h are less clear. Naively when m decreases the mass m_h seems huge in comparison and in this situation the decay is more probable. Thus the mean lifetime τ should decrease, i.e, Γ should increase. This is what happen when the variable is x_2 . The problem is that, as m decreases, the fermions of the triangle seem also to be heavier and heavier. In other words, the limit $m \rightarrow 0$ seems to be quite similar to take m_h and m_f very large, but with the quotient m_h/m_f fixed. So there is a compromise between the growing triangle mass, which decreases the probability, and the grow of m_h which seems to increase the probability. It is not clear which of the two effects will dominate at first sight. We argue that the right variable is x_1 . In fact a naive counting of how many times appears m_f in the numerator and denominator in $I(m, m_f, m_h)$ shows that there appears more times in the denominator. Since the integral is convergent it is reasonable, although not a proof, to state that $I(m, m_f, m_h)$ decreases with m_h . Of course the decay rate $\Gamma \sim m_h I^2$ should increase with m_h since the bigger the mass is, the more probable the decay is. But this condition does not invalidates a decreasing behavior of I . Based in these arguments, we choose the right variable as x_1 . If we assume $F(x_1)$ to be analytical, then the conditions $x_1 \ll 1$ and $F(0) = 0$ imply that

$$I(m, m_f, m_h) = F(x_1) \simeq c x_1^n,$$

with $n!c = F^{(n)}(0)$ the first non zero derivative of the function at zero. This follows up to corrections of higher order. In this situation

$$\Gamma \simeq \alpha_1 \alpha_2^4 m_h \left(\frac{m}{m_h}\right)^{2n} \left(\frac{m_h}{m_f}\right)^{2n\kappa}.$$

This is a crude estimation and we are assuming that the value of c is of the order of unity. A very large value of c would spoil this argument, but assuming that $c \sim 1$ we still have two non determined constants n and κ . Although m_h appears dividing in $I(m, m_f, m_h)$ we expect that Γ to decrease when

m_h decreases. The reason is that when this mass goes to very low values, the fermions of the triangle seem heavier and the probability of the decay should decrease. This means that $1 - 2n + 2nk > 0$, in this case m_h will be in the numerator in Γ . Since we are working with values $m_h < m_f$ but of the same order of magnitude, this expression becomes almost independent of k and the resulting decay width can be approximated as

$$\Gamma \simeq \alpha_1 \alpha_2^4 m_h \left(\frac{m}{m_h} \right)^{2n}.$$

Let us try to happen with successive powers. For $n = 1$ and the values

$$\alpha_1 \sim 1, \quad m \sim 1 \text{ MeV}, \quad m_h \sim 10^{13} \text{ GeV},$$

we see that if we choose an interaction between the hidden and ordinary sector $\alpha_2 \sim 10^{-6}$ it is obtained that

$$\Gamma \simeq 10^{-34} \text{ eV},$$

which corresponds to a lifetime $\tau \sim 10^{11}$ yrs. This is of the order or even larger than the estimated age of the universe. For higher powers the quantity $(m/m_h)^n$ is smaller and the lifetime is even larger, and one can introduce larger values for the coupling constant α_2 and still obtain the age of the universe.

In the previous calculation we have assumed that only one type of fermion is in the triangle. If we consider the presence of three fermions with different masses, then the dimensional argument given previously does not take place for I . But it is reasonable to argue that

$$I_{max} < I < I_{min},$$

with I_{max} and I_{min} the integrals corresponding to a triangle with the heavier and with the lightest eigenstates of mass. Let us consider for instance the component which is conformed by the left component of the neutrino. Since this component is not an eigenstate of mass, in fact $\nu_L \sim \nu_2 - 10^{-3}\nu_1$ one has to introduce a modified propagator $\frac{1}{p-m_{\nu_2}} - \frac{10^{-3}}{p-m_{\nu_1}}$, which gives

$$I_{\nu_L} = I_{1,1,1} - 3 \cdot 10^{-3} I_{1,1,2} + 3 \cdot 10^{-6} I_{2,2,1} - 10^{-9} I_{2,2,2},$$

where the indices indicate the eigenstates composing the triangle. Since the state with lower mass is ν_2 it follows that

$$I_{\nu_L} < (1 + 3 \cdot 10^{-3} + 3 \cdot 10^{-6} + 10^{-9}) I_{2,2,2} \simeq I_{2,2,2}.$$

Therefore

$$\Gamma_{\nu_L} < \Gamma_{2,2,2} \simeq 10^{-34} \text{ eV},$$

and the mean lifetime is even bigger than the age of the universe. Additionally the hidden electron has bigger mass than m_{ν_2} , thus the mayor contribution in the triangle is due to the hidden neutrino. The estimation we did for a triangle with a single fermion is then a valid lower bound of the mean lifetime.¹

6. Discussions

In the present work we presented a toy scenario describing simultaneously the actual vacuum energy density and long lived super-heavy dark matter. The model contains a doublet of fermions whose lagrangian possess an $SU(2)_L$ local symmetry. These fermions interacts with the ordinary matter with a very weak interaction. Both the fermions and the vector bosons acquire masses of the order of

¹We should comment that if our arguments about the functional dependence of I were wrong, then the true variable is x_2 and the resulting mean lifetime will be $\tau < 10^{-21}$ yrs, which is very small. Therefore in this case this particle does not give any dark matter at the present time. But we argue that this is not the case.

the GUT scale due to the Higgs mechanism. We have made a rough estimation of the mean lifetime of the hidden Higgs and we suggest that is of the order of, or even larger than, the age of the universe. This holds for a coupling constant $\alpha \sim 10^{-6}$ which, taking into account that the interaction may be even of gravitational order, is small but reasonable. The estimation of the mean life time of the Higgs relies on certain assumptions about the functional dependence of the decay width which are not fully justified. Therefore our statements are plausibility arguments instead of categorical affirmations. But if they are true, this hidden Higgs may be a component of the dark matter at the present times.

We have deliberately chosen the value of the hidden Higgs $m_h \sim 10^{13}\text{GeV}$, which is around the GKZ bound, to show in this case the lifetime is the age of the universe or bigger. Since no violations of this bound seem to take place, one needs a hidden Higgs with lifetime bigger than 10^{10} years in order to describe dark matter and simultaneously not to be seen by experiments. The mean lifetime of the hidden Higgs grows when its mass decreases which means that if the universe lifetime is achieved for this critical value $m_h \sim 10^{13}\text{GeV}$, the same will happen for particles below this bound. Note that the GKZ limit does not reject the possibility to have ultra massive particles in the present universe, as it involves a very large mass scale.

The presented scenario has also a right component of one of the fermions of the doublet and there is a global U(1) symmetry which is spontaneously broken giving also mass to this component. This fermion therefore acquires a mass matrix with two eigenvalues, one of the order of the GKZ bound and other with the order of the Planck mass. The current associated to the broken symmetry is anomalous due to an extremely weak interaction with ordinary matter. The associated axion acquires also a small mass component and its potential energy acts as an effective density of energy at the present times.

The presented model, as the majority of the quintessence models, assumes that the vacuum density energy is zero except for the contribution of the rolling scalar. In this model the vacuum energy is a temporary effect which disappear when the axion reach the minimum of the potential.

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